

Enrollment No: \_\_\_\_\_ Exam Seat No: \_\_\_\_\_

# C.U.SHAH UNIVERSITY

## Winter Examination-2021

Subject Name: Discrete Mathematics

Subject Code: 4TE04DSM2

Branch: B.Tech (CE)

Semester : 4

Date : 20/10/2021

Time : 11:00 To 02:00

Marks : 70

### Instructions:

- (1) Use of Programmable calculator and any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

**Q-1 Attempt the following questions: (14)**

- a) Find the least and greatest element in the poset  $\langle S_{30}, D \rangle$ , if they exist. (02)
- b) Define: Poset, Pseudo Graph (02)
- c) State Pigeonhole principle. (02)
- d) Find the atom and anti-atom of  $\langle S_{90}, D \rangle$ . (02)
- e) Prove that  $(ab + ab')a'b' = 0$ . (02)
- f)  $(Z_{11}, +_{11})$  is cyclic group.- True or False? (01)
- g) How many edges are there in a graph with 7 vertices each of degree 4? (01)
- h) How many Quantifiers are there? Name them. (02)

**Attempt any four questions from Q-2 to Q-8**

**Q-2 Attempt all questions. (14)**

- a) State and prove Stone's representation theorem. (10)
- b) Define: Semi group, Monoid, Subgroup (04)

**Q-3 Attempt all questions (14)**

- a) Show that  $\{0, 2, 4, 6\}$  is a subgroup of  $(Z_8, +_8)$ , where  $+_8$  is addition modulo 8. (05)
- b) Prove that  $\langle S_{35}, D \rangle$  is a complemented lattice and also draw the Hasse diagram of it. (05)
- c) Prove that  $\langle S_6, D \rangle$  is a sub lattice of  $\langle S_{30}, D \rangle$ . (04)

**Q-4 Attempt all questions (14)**

- a) Show that the set  $Q \setminus \{1\}$  is an abelian group with respect to the binary operation  $a * b = a + b - ab$ , for all  $a, b \in G$ . (07)



- b) Prove that  $1+2+3+\dots+n = \frac{n(n+1)}{2}$  by using mathematical induction. (07)

**Q-5 Attempt all questions** (14)

- a) For a lattice  $\langle P(\{a,b,c\}), \subseteq \rangle$ , answer the following questions: (07)

- i) Find cover of each element and draw the Hasse diagram.  
 ii) Find lower bound, upper bound, greatest lower bound, least upper bound of  $A = \{a,b\}$ .  
 iii) Find the least and greatest element of it.

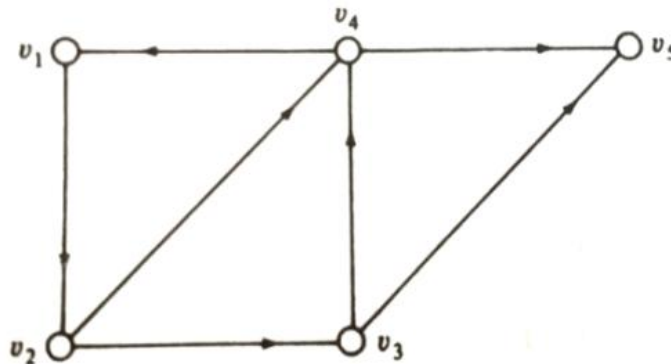
- b) Let  $\langle L, \leq \rangle$  be a lattice  $a, b \in L$  then prove that (07)

i)  $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$  ii)  $a \leq c \Leftrightarrow a \oplus (b * c) \leq (a \oplus b) * c$

**Q-6 Attempt all questions** (14)

- a) i) Draw the graph represented by given adjacency matrix (07)
- $$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 2 & 0 & 1 & 2 \\ 1 & 2 & 1 & 1 \\ 2 & 1 & 1 & 0 \end{bmatrix}$$

ii) Write the adjacency matrix from the given digraph.



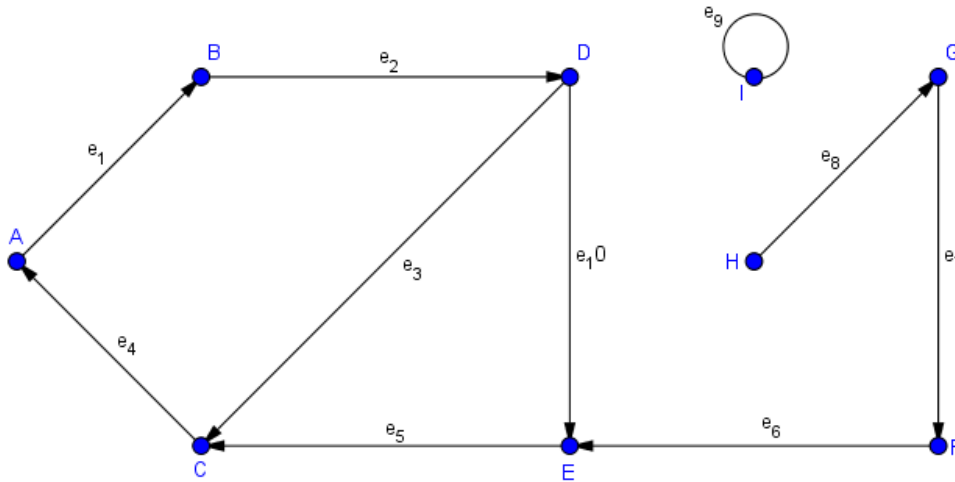
- b) State and prove Lagrange's theorem. (05)  
 c) Define: Propositional function. (02)

**Q-7 Attempt all questions.** (14)

- a) Obtain the sum of product canonical form of the Boolean expression in three variables  $\alpha(x, y, z) = (x * y)' * z$ . (05)
- b) Prove that  $(Z_8^*, \times_8)$  is a group. Is it commutative? (05)



c) Find all node base of the following diagram shown in the figure. (04)



**Q-8 Attempt all questions.** (14)

a) Define tree and draw a directed tree from following and also find the representation of binary tree. (07)  
 $(v_0(v_1(v_2)(v_3(v_4)(v_5))))(v_6(v_7(v_8))(v_9)(v_{10}))$

b) Do as directed: (07)

1) Translate the following in your own words.

$A(x)$ :  $x$  is a shark,  $B(x)$ :  $x$  is a fish,  $C(x)$ :  $x$  lives in water.

i)  $(\exists x)(B(x) \wedge \sim A(x))$       ii)  $(\forall x)(A(x) \vee C(x)) \Rightarrow B(x)$

2) Solve the recurrence relation  $a_n = 5a_{n-1} - 6a_{n-2}$ ,  $n \geq 2$ ;  $a_0 = 1$ ,  $a_1 = 2$ .

